Deliberative Voting

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ABSTRACT.   We analyze a model of jury decision making in which jurors deliberate before casting their votes. We consider a wide range of voting institutions and show that deliberations render these equivalent with respect to the sequential equilibrium outcomes they generate. In particular, in the context of a jury setup, all voting rules excluding the two types of unanimity rules (one requiring a unanimous consensus to acquit, one requiring a unanimous consensus to convict) induce the same set of equilibria outcomes. We show the robustness of our results with respect to several restrictions on communication protocols and jurors' strategies. Furthermore, we demonstrate that our observations extend to practically all of the voting structures commonly studied in the voting literature. The paper suggests the importance of accounting for communication in models of collective choice.

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1.   INTRODUCTION

1.1 OVERVIEW

Unanimous juries date back to fourteenth-century English common law and have become the American standard during the colonial period. Louisiana was the first to revolutionize the system. In 1928, it authorized a supermajority of 10 out of 12 jurors to acquit or convict in felony trials. Six years later, Oregon ensued with a section in its state constitution allowing for verdicts when five-sixths of a jury in a criminal trial agree. While murder defendants can most always be convicted solely by unanimous juries comprised of 12 jurors, civil cases in the U.S. portray an entirely different picture. 39 state courts have reduced the size of civil juries to 6, 7, 8, or 10 persons. Non-unanimous decision rules are utilized in 30 state courts and range from 2/3 majority, to 3/4 majority, to 5/6 majority, to 7/8 majority.[1]

The structure of jury decision making has long been the center of many political and legal debates (see Kalven and Zeisel [1966] and Hastie, Penrod, and Pennington [1983]). Recently, there has been a mushrooming of economic analysis of jury decision making, and more generally, of strategic voting behavior (see, e.g., Austen-Smith and Banks [1996], Myerson [1998], and Feddersen and Pesendorfer [1996, 1997, 1998]). The approach the economic literature has taken follows that of Condorcet [1785]. Broadly speaking, each juror receives an independent piece of information concerning the guilt of the defendant and subsequently casts her vote. Since the juror's vote matters only when she is pivotal, a strategic juror considers the information contained in the event of pivotality, taking into account her fellow jurors' strategies. There are two sets of conclusions this literature has produced. First, unanimity is expected to perform worse than non unanimous voting rules. Second, as jury size becomes infinitely large, non unanimous voting rules fully aggregate the available information and generate efficient outcomes.

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[1]See *State Court Organization 1998*, U.S. Department of Justice, Office of Justice Programs, available online at: http://www.ojp.usdoj.gov/bjs/pub/pdf/sco98.pdf

Surprisingly, two elements are missing from most of the prior theoretical discussion. The first is the fundamental fact of pre-vote deliberation. The second is the absence of comparison among non-unanimous voting rules (majority and different forms of supermajority), despite the prevalence of these rules in U.S. jurisdiction.[2]

The goal of the current paper is to explore the potential effects of deliberations on outcomes generated by a wide range of voting rules. We point to a broad array of environments, encompassing the standard jury setup, in which deliberations render a large class of (non unanimous) voting institutions equivalent in terms of the sets of (sequential) equilibrium outcomes they generate.[3]

We find our theoretical observations interesting in that they illustrate the importance of modeling deliberations in collective choice environments. In particular, any venture to explain the variance of different voting schemes observed in U.S. civil courts would necessitate structural assumptions on the protocols of jury deliberations.

In more detail, we start by considering a general version of the standard jury and strategic voting models. A jury of finite size faces a decision to acquit or convict a defendant. Each juror has a private type, corresponding to her private information and her personal preferences (e.g., her interpretation of what reasonable doubt means). We first concentrate on threshold voting rules. These are rules under which the defendant is convicted only if a certain threshold number of jurors favor conviction (corresponding to the rules prevailing in the U.S. court system). When communication is prohibited, different voting rules may generate different equilibrium outcomes (see Example 2). We investigate whether such differences between voting rules persist when the jurors can communicate with one another.

Our approach departs from the existing literature on deliberations and voting (Coughlan [2000], Austen-Smith and Feddersen [2002a,b], and Doraszelski, Gerardi and Squintani [2003], among others) in that we do not impose any restriction on the communication protocol. Indeed, there are many different plausible communication procedures. For example,

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[2]An exception is Persico [2004] who analyzes the optimal voting rule when private information is costly.

[3]An outcome specifies a random alternative (for example, acquittal or conviction) in every state of the world.

there can be one or several rounds of communication, individuals can exchange public or private messages, etc. In principle, different communication protocols may induce different outcomes. Thus, comparisons of voting rules may be sensitive to the particular specification of the communication procedure. Instead of focusing on a given protocol, in this paper we allow for any form of communication by formally adding a cheap talk stage prior to the voting stage. For any voting rule we consider the set of all outcomes that can be implemented with some procedure of communication.

Our first result shows that when deliberations occur, regardless of the information and preferences structure, all decision rules excluding the two types of unanimity rules (one requiring a unanimous consensus to acquit, one requiring a unanimous consensus to convict) generate the same set of equilibrium outcomes. Furthermore, under a unanimous voting rule it is possible to generate only a smaller set of outcomes.

To see the intuition behind this result, suppose an outcome is implementable under a certain voting rule. We show that the same outcome can be implemented with the following protocol and strategies. At the deliberation stage the jurors determine collectively the alternative to select. Then, they all vote in favor of it. In other words, the outcome is implemented with a strategy profile under which there is always unanimous consensus. Given these strategies, a juror does not have any incentive to deviate at the voting stage if the voting rule is non unanimous. In fact, her vote is never pivotal.

Consider the communication protocol and the voting strategies that we have just described. Of course, to determine whether an outcome is implementable or not, we need to consider the incentives of every juror to reveal her information during deliberations. These incentives depend on how the juror's information affects the collective choice. The incentives, however, do not depend on the voting rule. The equivalence result for non unanimous voting rules follows. Clearly, a unilateral deviation at the voting stage may be beneficial when unanimity is required. This is why unanimous rules yield smaller sets of equilibrium outcomes than those generated by intermediate voting rules.

The intuitive outline of the equivalence result suggests that a juror might find herself

voting (together with the rest of the jurors) for the alternative she least prefers, simply because her unilateral deviation will not alter the ultimate collective decision. Formally, this may raise the suspicion that the equivalence result hinges on jurors using weakly dominated strategies. On the contrary, we show that under rather weak restrictions on jurors' preferences, the equivalence result holds even when jurors do not use weakly dominated strategies. These restrictions rule out situations in which jurors are blind partisans of one of the alternatives and are keen to select that alternative regardless of the realization of types. This is in line with the process of *voir dire* exercised in the selection of American juries. Voir dire is used to eliminate jurors that may be unable to consider the case at hand fairly, based only on the evidence presented in court. Therefore, by and large, the restrictions we suggest should hold in practice. Furthermore, these restrictions are, in fact, satisfied by the bulk of the theoretical voting models analyzed in the literature.

As for generality, the equivalence result extends beyond its manifestation in the jury setup. In fact, the equivalence stretches directly to situations in which there are more than two alternatives. In Section 4 we consider all voting structures that are comprised of actions and voting rules. The action sets can be arbitrary (e.g., selected alternatives, ranking orders of all of the alternatives, etc.). The equivalence result holds as long as the voting rules in question are *veto-free.* That is, for any alternative there is a profile of feasible actions that yields that alternative via the voting rule, and is robust to any unilateral deviation. The class of veto-free voting rules contains most of the voting rules discussed in the literature (see, e.g. Cox [1987]). For example, plurality rule, Borda rule, and Condorcet winner's selection method (for more than two alternatives) are veto-free. Specifically, we show that for any fixed set of alternatives, all veto-free voting structures are equivalent with respect to the equilibrium outcomes they generate. Furthermore, non veto-free structures yield sets of equilibrium outcomes that are subsets of those corresponding to the veto-free structures.

The rest of the paper is structured as follows. Section 1.2 overviews the related literature. Section 2 describes the general setup of a deliberating jury's decision making and provides the preliminary comparison between different threshold voting rules. Section 3 specifies the

robustness of the equivalence results with respect to the communication protocols (Section 3.1) and with respect to the jurors' strategies (Section 3.2). Section 4 demonstrates the generalized equivalence result pertaining to any set of alternatives. Section 5 concludes. Most proofs are relegated to the Appendix.

1.2 Literature Review

While the literature on strategic voting with communication is still in its inception, there are a few recent contributions that relate to the current paper. Coughlan [2000] adds a straw poll preceding the voting stage in a private information, two alternative environment. He shows that voters reveal their information truthfully if and only if their preferences are sufficiently close. Austen-Smith and Feddersen are among the first to put forward different models of deliberative committees. Austen-Smith and Feddersen [2002a] consider committees of three agents who need to choose one of two alternatives. Each agent has private information on two dimensions: perfect information concerning her preferences and noisy information concerning the state of the world. They model deliberations as a one-round process in which all agents simultaneously send public messages. The restriction to this particular form of communication allows them to consider an equilibrium concept (reminiscent of trembling hand perfection) stronger than the notion we use (sequential equilibrium in weakly undominated strategies). They show that when such deliberations precede the voting stage, majority rule induces more information sharing and fewer decision-making errors than unanimity. Austen-Smith and Feddersen [2002b] look at a similar environment in which any number of agents can publicly send arbitrary messages before casting their votes. They provide conditions under which unanimity cannot induce full revelation of private information in equilibria comprised of weakly undominated strategies. Furthermore, if full revelation is possible under unanimity, then it is possible under any other rule. Doraszelski, Gerardi and Squintani [2003] study a two-player model with communication and voting. Preferences are heterogenous (not necessarily aligned) and private information. They show that some, but not all, information is transmitted in equilibrium, and that communication is beneficial.

In a similar vein, there has been some experimental work on voting with communication. Guarnaschelli, McKelvey, and Palfrey [2000] constructed an experiment replicating Coughlan's [2000] setup. They noted that during deliberations, voters tend to expose their private information but not to the full extent as predicted by Coughlan's [2000] results. Recently, Goeree and Yariv [2006] conducted an array of experiments testing for the effects of free form communication on jury outcomes. Their observations provide support for the results reported in this paper.

Blinder and Morgan [2005] conducted a conceptually different experiment in which groups were required to solve two problems - a statistical urn problem and a monetary policy puzzle. The groups could converse before casting their votes using either majority rule or unanimity. They found no significant difference in the decision lag when group decisions were made by majority rule relative to when they were made under a unanimity requirement.

The idea that communication may render a class of institutions equivalent appears in Matthews and Postlewaite [1989] who compare all two-person double auctions and show that they generate the same sets of equilibrium outcomes when the bidders can communicate before submitting their bids.

2. Deliberative Voting with Two Alternatives

2.1 Setup

A jury of $n \geqslant 3$ members has to choose one of two alternatives: A (acquit) or C (convict).[4] Each juror i has a type t_i which is private information. The type t_i can capture juror i's preferences, her private knowledge of each of the alternatives' consequences, etc. We let T_i denote the set of types of juror i, and assume that it is finite. $T = \prod_{i=1}^{n} T_i$ denotes the set of profiles of types, and p is the probability distribution over T. To simplify the exposition and avoid a few technical issues, we assume that the probability distribution p has full support

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[4]While our terminology follows closely that of the standard jury models, of course the same setup can be used for any environment in which a committee selects one of two alternatives. Section 4 extends the analysis to more general voting environments (more than two alternatives, and arbitrary voting rules).

(that is, $p(t) > 0$ for every type profile t in T).[5] A juror's utility depends on the profile of types and the chosen alternative. Formally, for each juror i there exists a utility function $u_i : \{A, C\} \times T \to \mathbb{R}$.

This setup encompasses the strategic voting models that have been commonly analyzed in the literature (see, for instance, Austen-Smith and Banks [1996] or Feddersen and Pesendorfer [1998]). We shall often use (a simple version of) the standard voting setup to illustrate our results, as described in the example below.

Example 1 (the standard jury setup) A jury of n individuals has to determine the fate of a defendant. There are two states, I (the defendant is innocent) and G (the defendant is guilty), with prior probabilities $P(I)$ and $P(G)$, respectively.

Juror i's preferences are given by:

$$\widehat{u}_i(C, G) = \widehat{u}_i(A, I) = 0, \quad \widehat{u}_i(A, G) = -(1 - q_i), \quad \widehat{u}_i(C, I) = -q_i,$$

where $q_i \in (0, 1)$ denotes juror i's threshold of reasonable doubt (capturing her concern for convicting the innocent relative to that for acquitting the innocent).

Each agent i observes a signal $t_i \in \{\mathcal{I}, \mathcal{G}\}$ of accuracy π. That is,

$$\Pr(t_i = \mathcal{I}|I) = \Pr(t_i = \mathcal{G}|G) = \pi.$$

Conditional on the state, signals are independent across jurors. Let $T = \{\mathcal{I}, \mathcal{G}\}^n$ denote the set of profiles of signals.

Clearly, the utility function $u_i : \{A, C\} \times T \to \mathbb{R}$ of juror i is given by:

$$u_i(A, t) = -(1 - q_i)\Pr(G|t),$$
$$u_i(C, t) = -q_i \Pr(I|t).$$

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[5]Most of our results do not depend on the full support assumption. In particular, Propositions 1 and 2 hold for any probability distribution.

Note that the setup studied in this paper is more general than that of the standard model in that types can be taken from arbitrary sets, and there are essentially no restrictions on the prior distribution over types. In particular, types may be correlated across individuals.

As for the choice of the defendant's fate, jurors select an alternative by voting. Each juror can vote to acquit, a, or to convict, c.[6] We let $V_i = \{a, c\}$ denote the set of actions available to juror i, and $V = \{a, c\}^n$ the set of action profiles. Under the voting rule $r = 1, \ldots, n$, the alternative C is selected if and only if r or more jurors vote to convict.

Given a voting rule r and a profile of votes v, we let $\psi_r(v)$ denote the jury's decision. Formally, $\psi_r : V \to \{A, C\}$ is defined as follows:

$$\psi_r(v) = \begin{cases} A & \text{if } |\{i : v_i = c\}| < r, \\ C & \text{if } |\{i : v_i = c\}| \geqslant r. \end{cases}$$

The voting rule r defines the following voting game G_r. Nature selects a profile of types in T according to the probability distribution p, then jurors learn their types, after which they vote simultaneously. If the profiles of types and actions are t and v, respectively, juror i obtains $u_i(\psi_r(v), t)$.[7]

In general, the set of equilibrium outcomes corresponding to the game G_r does not coincide with the set of equilibrium outcomes corresponding to the game $G_{r'}$, where $r \neq r'$, as the following example illustrates.

Example 2 Consider the standard jury setup with $n = 4$. Assume that the two states are equally likely, $P(I) = P(G)$, and that the accuracy of the signal is $\pi = 3/4$. Lastly, suppose that $q_1 = q_2 = 2/5$ and $q_3 = q_4 = 3/5$.

Let $\beta(k', k)$ denote the posterior probability that the defendant is guilty when k' out of k jurors observe the guilty signal $\mathcal{G}$. It is easy to check that $\beta(1, 4) = 1/10$, $\beta(2, 4) = 1/2$ and $\beta(3, 4) = 9/10$. It follows that jurors 1 and 2 require only two of the four signals to indicate guilt in order to find the defendant culpable, whereas jurors 3

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[6]The case in which jurors can also choose to abstain is covered by the analysis provided in Section 4.

[7]Note that, as is standard in the voting literature, we assume that a juror's utility depends on the outcome of the vote, but not on the particular vote profile that generated the outcome.

and 4 require at least three guilty signals. Notice that there is disagreement among the jurors when exactly three signals indicate guilt.

Consider the voting game G_2 and the following strategy profile. The first three jurors vote informatively (i.e., they vote to acquit if they observe $\mathcal{I}$ and to convict if they observe $\mathcal{G}$), whereas juror 4 votes to acquit regardless of her signal. This strategy profile constitutes a (Bayesian Nash) equilibrium of G_2. First, consider juror $i = 1, 2, 3$. The probability that the defendant is guilty given that juror i observes signal t_i and her vote is pivotal is equal to

$$\Pr\left(G|t_i \text{ and } i \text{ is pivotal}\right) = \begin{cases} \beta\left(1,3\right) = 1/4 < q_i & \text{if } t_i = \mathcal{I} \\ \beta\left(2,3\right) = 3/4 > q_i & \text{if } t_i = \mathcal{G} \end{cases}$$

Clearly, juror i has an incentive to vote according to her signal. For juror 4 we have

$$\Pr\left(G|t_4 \text{ and } 4 \text{ is pivotal}\right) = \begin{cases} \beta\left(1,4\right) = 1/10 < q_4 & \text{if } t_4 = \mathcal{I} \\ \beta\left(2,4\right) = 1/2 < q_4 & \text{if } t_4 = \mathcal{G} \end{cases}$$

In both cases, juror 4 prefers to vote to acquit.

The above equilibrium implements the outcome $\theta : \{\mathcal{I}, \mathcal{G}\}^4 \to \Delta\{A, C\}$ under which the alternative C is chosen if at least two of the first three jurors receive the signal $\mathcal{G}$.

However, θ is not an equilibrium outcome of the voting game G_3. In fact, under the voting rule $r = 3$, θ can be implemented only if the first three jurors vote informatively and juror 4 votes to *convict* regardless of her signal. But this strategy profile does not constitute an equilibrium of G_3. In particular, consider juror 4 and suppose that she observes signal $\mathcal{I}$. The probability that the defendant is guilty when her vote is pivotal is equal to $\beta\left(2,4\right) = 1/2 < q_4$. It follows that juror 4 has an incentive to deviate and vote to acquit.

The focus of the paper is the comparison of different voting rules when jurors are allowed

to deliberate before casting their votes. We model deliberations by adding cheap talk to the voting game G_r. A cheap talk extension of G_r is an extensive-form game in which the jurors, after learning their types, exchange messages. At the last stage of the game, the jurors vote. Payoffs depend on the jurors' types and votes, but not on their messages. For technical simplicity, it will be useful to start by assuming that that there exists an impartial mediator who helps the jurors communicate. This assumption will be dropped later on in Section 3.1.[8]

A strategy profile σ of a cheap talk extension of G_r induces an outcome, i.e., a mapping γ_σ from the set of types T into the interval $[0,1]$. $\gamma_\sigma(t)$ denotes the probability that the defendant is convicted when the profile of types is t (and the jurors adopt the strategy profile σ). We let Γ_r denote the set of outcomes induced by sequential equilibria of cheap talk extensions of G_r.

The notion of communication equilibrium (Myerson [1982], Forges [1986]) allows us to characterize the set Γ_r. As is standard, we denote by $\Delta(V)$ the set of probability distributions over the set of action profiles V. A mapping μ from T into $\Delta(V)$ is a communication equilibrium of G_r if and only if the following inequalities hold:[9]

$$\sum_{t_{-i}\in T_{-i}} p(t_{-i}|t_i) \sum_{v\in V} \mu(v|t) u_i(\psi_r(v),t) \geqslant \sum_{t_{-i}\in T_{-i}} p(t_{-i}|t_i) \sum_{v\in V} \mu(v|t_{-i},t_i') u_i(\psi_r(v_{-i},\delta(v_i)),t)$$

$$\forall i = 1,\ldots,n, \quad \forall (t_i,t_i') \in T_i^2, \quad \forall \delta : \{a,c\} \rightarrow \{a,c\}.$$

$$(1)$$

Intuitively, consider the following cheap talk extension of the voting game G_r. All jurors report their types to the mediator. Suppose that the profile of reports is t. Then the mediator selects an action profile randomly, according to the probability distribution $\mu(t)$, and informs each juror of her own action. Finally, the jurors vote. The inequalities in (1) guarantee that the cheap talk extension admits a sequential equilibrium in which two types

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[8]For a formal definition of cheap talk extensions to arbitrary games see Myerson [1991].

[9]As usual, T_{-i} denotes the set of types of players other than i.

of incentive constraints hold: 1. all jurors are sincere; and 2. all jurors obey the mediator's recommendations.

It follows from the argument above that any communication equilibrium induces an outcome in Γ_r. The revelation principle (Myerson [1982]) assures that Γ_r contains, in fact, only outcomes that are induced by communication equilibria.[10] In particular, the set Γ_r coincides with the set of outcomes induced by communication equilibria of G_r. Let V_r^C denote the set of profiles of votes that lead to conviction under the voting rule r. Formally, $V_r^C = \{v \in V : \psi_r(v) = C\}$. Then we have:

$$\Gamma_r = \{\gamma : T \to [0,1] \,|\, \exists \text{ a communication equilibrium } \mu \text{ of } G_r$$
$$\text{such that } \gamma(t) = \sum_{v \in V_r^C} \mu(v|t) \text{ for every } t \in T\}.$$

2.2 A PRELIMINARY RESULT

The goal of this section is to compare different voting rules when the jurors can communicate. We start with a simple example which suggests that communication may render two different voting rules similar in terms of the outcomes that they implement.

Example 3 Consider the environment of Example 2. Recall that θ denotes the outcome under which the defendant is convicted if at least two of the first three jurors observe the signal $\mathcal{G}$. Recall also that θ is an equilibrium outcome of the voting game G_2 but not of G_3.

We now show that θ can be also implemented with the voting rule $r = 3$ when communication is allowed. Consider the following communication protocol. All jurors report their signal to the mediator, who then dispels unanimous recommendations that implement θ. That is, the mediator recommends the action c to every juror if at least two

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[10]Indeed, the original revelation principle extends directly to sequential (rather than Bayesian) equilibria in our setup under the assumption of full support of the prior distribution p.

of the first three jurors report signal $\mathcal{G}$. In all other cases, the mediator recommends the action a to every juror.

The strategy profile in which all jurors are truthful and obedient is a sequential equilibrium of the voting game with communication. Notice that the mediator ignores the signal of juror 4 and that θ represents the optimal outcome for all jurors when only three signals are available. It follows that for juror $i = 1, 2, 3$ it is strictly optimal to play the equilibrium strategy (see Example 2). Finally, consider juror 4. First, suppose that she receives the recommendation a. The posterior probability that the defendant is guilty is equal to

$$\Pr\left(G | t_4 \text{ and at least two other jurors observed } \mathcal{I}\right) = \begin{cases} 5/86 < q_4 & \text{if } t_4 = \mathcal{I} \\ 5/14 < q_4 & \text{if } t_4 = \mathcal{G} \end{cases},$$

Suppose now that juror 4 receives the recommendation c. The posterior probability that the defendant is guilty is equal to

$$\Pr\left(G | t_4 \text{ and at least two other jurors observed } \mathcal{G}\right) = \begin{cases} 9/14 > q_4 & \text{if } t_4 = \mathcal{I} \\ 81/86 > q_4 & \text{if } t_4 = \mathcal{G} \end{cases},$$

It follows that juror 4 always has a strict incentive to obey the mediator's recommendation.

Thus, while unilateral deviations are inconsequential at the voting stage of our game with communication, the jurors are, in fact, using undominated strategies. This observation will be expanded on in Section 3.2.

We are now ready to generalize the above example and compare the sets Γ_r and $\Gamma_{r'}$ for two different voting rules r and r'. In Proposition 1 we show that, except for the voting rules $r = 1$ and $r = n$,[11] all other "intermediate" rules are equivalent. If jurors can communicate,

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[11]The voting rules $r = 1$ and $r = n$ are the only rules which require a unanimous consensus in order to adopt a certain alternative (A if $r = 1$, C if $r = n$).

every outcome that can be implemented with a voting rule $r \neq 1, n$ can also be implemented with a different voting rule $r' \neq 1, n$. Furthermore, by adopting an extreme voting rule ($r = 1$ or $r = n$), we cannot enlarge the set of equilibrium outcomes.

Proposition 1 $\Gamma_2 = \ldots = \Gamma_{n-1}$. *Moreover,* $\Gamma_1 \subseteq \Gamma_2$ *and* $\Gamma_n \subseteq \Gamma_2$, *and these inclusions may be strict.*

Proof:

Consider any voting rule $r = 1, \ldots, n$. The first step of our proof is to show that if γ belongs to Γ_r then γ satisfies the following inequalities:

$$\sum_{t_{-i} \in T_{-i}} p\left(t_{-i} | t_i\right) \left[\gamma\left(t\right) u_i\left(C, t\right) + \left(1 - \gamma\left(t\right)\right) u_i\left(A, t\right)\right] \geqslant$$

$$\sum_{t_{-i} \in T_{-i}} p\left(t_{-i} | t_i\right) \left[\gamma\left(t_{-i}, t_i'\right) u_i\left(C, t\right) + \left(1 - \gamma\left(t_{-i}, t_i'\right)\right) u_i\left(A, t\right)\right] \tag{2}$$

$$\forall i = 1, \ldots, n, \quad \forall \left(t_i, t_i'\right) \in T_i^2.$$

If γ is in Γ_r, there exists a communication equilibrium μ of G_r that induces γ. For every juror i and for every pair $\left(t_i, t_i'\right)$ we therefore have:

$$\sum_{t_{-i} \in T_{-i}} p\left(t_{-i} | t_i\right) \left[\gamma\left(t\right) u_i\left(C, t\right) + \left(1 - \gamma\left(t\right)\right) u_i\left(A, t\right)\right] =$$

$$\sum_{t_{-i} \in T_{-i}} p\left(t_{-i} | t_i\right) \left[\left(\sum_{v \in V_r^C} \mu\left(v | t\right)\right) u_i\left(C, t\right) + \left(1 - \sum_{v \in V_r^C} \mu\left(v | t\right)\right) u_i\left(A, t\right)\right] =$$

$$\sum_{t_{-i} \in T_{-i}} p\left(t_{-i} | t_i\right) \sum_{v \in V} \mu\left(v | t\right) u_i\left(\psi_r\left(v\right), t\right) \geqslant \sum_{t_{-i} \in T_{-i}} p\left(t_{-i} | t_i\right) \sum_{v \in V} \mu\left(v | t_{-i}, t_i'\right) u_i\left(\psi_r\left(v\right), t\right) =$$

$$\sum_{t_{-i} \in T_{-i}} p\left(t_{-i} | t_i\right) \left[\left(\sum_{v \in V_r^C} \mu\left(v | t_{-i}, t_i'\right)\right) u_i\left(C, t\right) + \left(1 - \sum_{v \in V_r^C} \mu\left(v | t_{-i}, t_i'\right)\right) u_i\left(A, t\right)\right] =$$

$$\sum_{t_{-i} \in T_{-i}} p\left(t_{-i} | t_i\right) \left[\gamma\left(t_{-i}, t_i'\right) u_i\left(C, t\right) + \left(1 - \gamma\left(t_{-i}, t_i'\right)\right) u_i\left(A, t\right)\right],$$

where the inequality follows from the truth-telling constraints of the communication equilibrium μ.

Consider now a voting rule $r = 2, \ldots, n-1$. We demonstrate that if $\gamma : T \to [0, 1]$ satisfies inequality (2), then γ belongs to Γ_r. Given γ, consider the following mapping $\tilde{\mu}$ from T into

$\Delta (V)$:

$$\tilde{\mu} (v|t) = \begin{cases} \gamma (t) & \text{if } v = (c, \ldots, c), \\ 1 - \gamma (t) & \text{if } v = (a, \ldots, a), \\ 0 & \text{otherwise.} \end{cases}$$

Obviously, $\tilde{\mu}$ induces γ. It is easy to show that $\tilde{\mu}$ is a communication equilibrium of G_r. First of all, no juror has an incentive to disobey the mediator's recommendation. Indeed, when the mediator follows $\tilde{\mu}$, she makes the same recommendation to all jurors. A juror's vote cannot change the final outcome if all her fellow jurors are obedient (notice that we are not considering $r = 1$ and $r = n$). Furthermore, the fact that γ satisfies inequality (2) implies that no juror has an incentive to lie to the mediator when her fellow jurors are sincere.

We conclude that $\Gamma_2, \ldots, \Gamma_{n-1}$ coincide with the set of the mappings from T into $[0, 1]$ which satisfy inequality (2). Moreover, this set contains Γ_1 and Γ_n.

We now show that the inclusions $\Gamma_1 \subseteq \Gamma_2$ and $\Gamma_n \subseteq \Gamma_2$ may be strict. Indeed, consider first the unanimity rule $r = n$. Any outcome in Γ_n can be implemented with a communication equilibrium in which the mediator sends the same recommendation to all jurors. This guarantees that no juror has an incentive to disobey recommendation a. Of course, the action profile $(c, \ldots, c)$ is necessary to convict the defendant under the unanimity rule. If a juror does not follow recommendation c the final decision will be A (in this case the juror's message is irrelevant). Thus, an outcome $\gamma : T \to [0, 1]$ satisfies the obedience constraints if and only if the following inequalities hold:

$$\sum_{t_{-i} \in T_{-i}} p (t_{-i}|t_i) [u_i (C, t) - u_i (A, t)] \gamma (t) \geqslant 0$$

$$\forall i = 1, \ldots, n, \quad \forall t_i \in T_i. \tag{3}$$

We conclude that the set Γ_n consists of all mappings from T into $[0, 1]$ which satisfy inequalities (2) and (3). Similarly, Γ_1 coincides with the set of mappings from T into $[0, 1]$

which satisfy inequality (2) and the following inequality:

$$\sum_{t_{-i}\in T_{-i}} p\left(t_{-i}|t_i\right)\left[u_i\left(A,t\right)-u_i\left(C,t\right)\right]\left(1-\gamma\left(t\right)\right)\geqslant 0$$

(4)

$$\forall i=1,\ldots,n,\quad \forall t_i\in T_i.$$

Clearly, inequality (2) implies neither inequality (3) nor inequality (4). In particular, there may exist outcomes that belong to Γ_2 but do not belong to Γ_1 or to Γ_n. ∎

The proof of Proposition 1 is rather intuitive. Consider an outcome implementable with deliberations under the voting rule $r = 2,\ldots,n-1$. As we pointed out in Section 2.1, this outcome can be implemented with a communication protocol (a cheap talk extension) in which the jurors truthfully reveal their types to an impartial mediator who disperses recommendations to all jurors. Each profile of recommended actions corresponds, through r, to one of the two social alternatives. Consider then a modification of this protocol which prescribes to each profile of private reports a unanimous recommendation to the jurors matching the social alternative that would have resulted in the original protocol. Since $1 < r < n$, any unilateral deviation will not alter the outcome, and so equilibrium incentives are maintained. In particular, the modified protocol generates an implementable outcome coinciding with the one we started with. Moreover, since all recommendations are unanimous, this remains an equilibrium outcome for any voting rule $r' = 2,\ldots,n-1$. The equivalence of all intermediate threshold rules follows.

In fact, in the proof of Proposition 1 we derive the full characterization of equilibria sets corresponding to rules $r = 2,...,n-1$, which is given by the set of inequalities (2).

With voting rules 1 and n (unanimity) it is generally possible to implement only a subset of the outcomes that can be implemented with the "intermediate" voting rules $r = 2,..,n-1$. Intuitively, consider first the unanimity rule $r = n$. Any outcome generated with unanimity can be implemented via a mediator who dispels unanimous recommendations as above. Just like all of the intermediate voting rules, when the recommendation is for everyone to vote for acquittal, no one juror has an incentive to deviate, since her deviation cannot affect the final

outcome. However, when the recommendation is to cast a vote for conviction, a unilateral deviation can in fact alter the final decision under unanimity, and an additional constraint needs to be satisfied for jurors to obey such a recommendation. This supplementary condition identifies outcomes corresponding to unanimity as a subset of the outcomes generated by any of the intermediate voting rules. Similar intuition holds for the inclusion of outcomes generated by voting rule $r = 1$. Formally, these additional constraints are derived in the proof and given by inequalities (3) and (4).

3.   Restricting Protocols and Strategies

In this section, we illustrate the robustness of our preliminary result. First, we show that the equivalence does not require extremely complex communication protocols. In fact, restricting communication to take place publicly in one round, maintains the equivalence, even when there is no mediating party. Second, we demonstrate the robustness of the result to stronger assumptions on voters' behavior, crossing out the potential use of weakly dominated strategies.

3.1 Public Communication

Throughout Section 2 we have assumed that each juror can communicate privately with a trustworthy mediator. However, in many group decisions, particularly in juries, a reliable mediator is not available and the agents can only exchange messages with each other. In addition, there are cases, like jury deliberations, in which an individual cannot communicate with only *some* of her fellow decision makers. In juries, during deliberations communication is public and every message is observed by *all* the jurors. In this section we investigate how these restrictions affect our results.

Given a voting rule r, we define a cheap talk extension with public communication of G_r as follows. After learning their types, the jurors undergo a finite number of rounds of communication. In each round one or more individuals send a message to all jurors. At the end of the communication phase, the jurors cast their votes simultaneously and the chosen alternative is C (i.e., the defendant is convicted) if r or more jurors vote c (convict).

Notice that outcomes induced by sequential equilibria of cheap talk extensions with public communication of G_r are included in Γ_r, $r = 1, \ldots, n$. In fact, any outcome that can be implemented without a mediator can be also implemented with a mediator. Proposition 2 illustrates that the opposite inclusion holds for $r = 2, \ldots, n-1$, even when the public communication phase is restricted to only one round.

Proposition 2 *Consider the voting rule $r = 2, \ldots, n-1$. Any outcome in Γ_r can be implemented (in sequential equilibrium) with one round of public communication.*

In the proof of Proposition 2 we show that any outcome in Γ_r, $r = 2, \ldots, n-1$, can be implemented with a communication protocol in which the jurors publicly announce their types. In addition, two jurors (say jurors 1 and 2) also announce two real numbers in the unit interval. These numbers allow the jurors to coordinate their votes. In equilibrium, all jurors truthfully reveal their information. Jurors 1 and 2 choose their numbers randomly, according to the uniform distributions. The two numbers generate a jointly controlled lottery that determines the final outcome (A or C) as a function of the announced types.[12] At the voting stage all jurors vote in favor of the chosen alternative.

Notice that the specific communication protocol introduced in the proof of Proposition 2 could be used to implement the entire set of sequential equilibrium outcomes, regardless of the (intermediate) threshold voting rule. This result is reminiscent of the construction introduced by Forges [1990], in which one *universal mechanism* serves to implement the equilibrium outcomes of all noncooperative games with incomplete information and at least four players. This observation is important from the point of view of mechanism design. Indeed, consider a designer who aims at implementing a certain feasible outcome. To accomplish this, the designer should do two things. First, she should find a cheap talk extension with an equilibrium that induces the desired outcome. Second, the designer should induce the agents to play that equilibrium. Our analysis shows that, without loss of generality, the

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[12]A jointly controlled lottery is a communication procedure that allows two or more players to select an outcome randomly according to a given probability distribution. The lottery is robust to unilateral deviations. See the proof of Proposition 2 for details.

designer can use the communication protocol described in the proof of Proposition 2 and restrict attention to the problem of inducing agents to play the desired equilibrium.[13]

An immediate consequence of Proposition 2 is that the results in Proposition 1 extend to the case of public communication. Suppose that a reliable mediator is not available and the jurors can only exchange public messages. Then all intermediate voting rules are equivalent. Moreover, any outcome that can be implemented with the voting rules $r = 1, n$ can be also implemented with the intermediate voting rules.

3.2 WEAKLY UNDOMINATED STRATEGIES

Consider a jury with twelve members. Suppose that the jurors do not have any private information. Furthermore, nine jurors prefer to acquit the defendant and three jurors prefer to convict her.[14] According to Propositions 1 and 2, if the jurors can communicate, all intermediate voting rules implement the same set of outcomes. In particular, any probability distribution between acquittal and conviction is a sequential equilibrium outcome under the voting rules $r = 2, \ldots, 11$.

Suppose now that the jurors do not use weakly dominated strategies. Are all intermediate voting rules still equivalent? It is not difficult to see that the answer is negative. Consider a juror who prefers acquittal. Clearly, any strategy under which the juror votes to convict is weakly dominated. If her vote is not pivotal the juror is indifferent between voting to acquit and to convict. On the other hand, if her vote is pivotal the juror has a strict incentive to vote to acquit. Similarly, a juror who prefers conviction and does not use weakly dominated strategies will never vote to acquit. In this simple example communication does not play any role. Under the voting rule $r = 1, \ldots, 3$ ($r = 4, \ldots, 12$) the equilibrium outcome is unique and the defendant is convicted (acquitted) with probability one.

Certainly, if one is willing to entertain the idea that individuals care about their vote

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[13]See Gerardi and Yariv [2005] for an analysis of a particular mechanism design problem of this form.

[14]In this example we assume that the jurors have "partisan" preferences. Of course, partisan preferences are more appropriate in environments different from juries (for example, an election with two candidates). Here we follow our terminology and frame the example in the context of the jury model.

coinciding with the selected alternative, then our equivalence result holds directly with un-dominated strategies. For example, suppose that every juror has the following lexicographic preferences. If her vote affects the final outcome, the juror's preferences are identical to the preferences described in Section 2.1. If her vote does not influence the final outcome, the juror prefers to vote in favor of the winning alternative. It is easy to check that the strategies employed in the proof of Proposition 2 are weakly undominated.

In this section we take a different route to ruling out weakly dominated strategies. Namely, we show that under a few weak assumptions on the preference and information structure, the results of Proposition 1 extend to the case of weakly undominated strategies. In particular, we show that any outcome in Γ_r, $r = 2, \ldots, n-1$, can be implemented with a cheap talk extension and a sequential equilibrium that does not use dominated strategies.

To simplify the exposition and the proof of Proposition 3 below, we assume the presence of an impartial mediator who helps the jurors communicate.[15] For any voting rule r, we consider cheap talk extensions in which the jurors communicate with one another and with the mediator.

In games with incomplete information (as the ones we are considering) there are two different notions of dominance: *ex-ante dominance* and *interim dominance* (see Fudenberg and Tirole [1991], pages 226-229). Ex-ante domination requires that all types of a juror have the same beliefs about the play of the other jurors. In contrast, interim domination allows different types to have different beliefs. As is well known, it is easier for a strategy to be interim weakly undominated than ex-ante weakly undominated. In this section we restrict attention to the stronger notion of ex-ante undominated strategies.

Formally, consider a voting rule r and cheap talk extension of G_r. Let $\sigma_i = (\sigma_i(t_i))_{t_i \in T_i}$ denote a (possibly mixed) strategy of juror i, where $\sigma_i(t_i)$ indicates the actions that juror i chooses when her type is t_i. By a slight abuse of notation, we will extend the domain of u_i and let $u_i(\sigma_i(t_i), \sigma_{-i}(t_{-i}), t)$ denote the expected utility of juror i when the profile of types

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[15]The equivalence result with weakly undominated strategies holds even when a mediator is not available. See our discussion following Proposition 3 below.

is t and the jurors use the strategies $(\sigma_i(t_i), \sigma_{-i}(t_{-i}))$.

Definition 1 *The strategy σ_i is ex-ante weakly undominated for juror i if there does not exist a strategy $\hat{\sigma}_i$ such that*

$$\sum_{t_i} p(t_i) \sum_{t_{-i}} p(t_{-i} \mid t_i) \left[u_i(\hat{\sigma}_i(t_i), \sigma_{-i}(t_{-i}), t) - u_i(\sigma_i(t_i), \sigma_{-i}(t_{-i}), t) \right] \geqslant 0,$$

for every strategy profile σ_{-i} and with at least one strict inequality.

We let $\hat{\Gamma}_r$ denote the sets of outcomes induced by sequential equilibria of cheap talk extensions of G_r in which the jurors do not use ex-ante weakly dominated strategies. In order to characterize the set $\hat{\Gamma}_r$, we need to make the following assumptions.

A1 (Informational Smallness) For every juror $i = 1, \ldots, n$ and every alternative $x = A, C$, there exists a profile of types t_{-i}^x in T_{-i} such that $u_i\left(x, t_i, t_{-i}^x\right) > u_i\left(y, t_i, t_{-i}^x\right)$,for every $t_i \in T_i, y \neq x$.

A2 (Informational Significance) For every juror $i = 1, \ldots, n$ and every pair of types t_i, t_i' in T_i, there exists a profile of types $t_{-i}^{\left(t_i, t_i'\right)}$ in T_{-i} such that

$$u_i\left(x, t_i, t_{-i}^{\left(t_i, t_i'\right)}\right) > u_i\left(y, t_i, t_{-i}^{\left(t_i, t_i'\right)}\right),$$

$$u_i\left(y, t_i', t_{-i}^{\left(t_i, t_i'\right)}\right) > u_i\left(x, t_i', t_{-i}^{\left(t_i, t_i'\right)}\right),$$

where $x, y = A, C$ and $x \neq y$.

Assumption A1 guarantees that each juror would benefit from the information available to the other jurors. In some sense, this assumption implies that each juror is "information-ally small".[16] A juror's information is not sufficient for her to conclude which alternative

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[16]Note that A1 is qualitatively different from the concept of informational smallness introduced by McLean and Postlewaite [2002] in that it is not probabilistic. In fact, for any i, the probability the realized types satisfy A1 can be arbitrarily close to 0.

is optimal. Intuitively, Assumption A1 is crucial for the equivalence result to hold with undominated strategies, since it allows us to rule out those situations in which some jurors always prefer one of the alternatives over the other, regardless of the realized types of her fellow jurors.

On the other hand, assumption A2 ensures that a juror's information is never useless. There is always a situation in which two types of the same juror prefer different alternatives.

In practice, the process of *voir dire* exercised in the American court system is effectively intended to assure that assumptions A1 and A2 hold for trial jurors. Indeed, voir dire is used to select jurors that are sufficiently unbiased so as to consider the case at hand fairly, based only on the evidence presented in court. Technically, assumptions A1 and A2 are satisfied by most models studied in the literature. The following example illustrates the restrictions they impose on the standard jury model.

Example 4 Consider the standard jury setup with n jurors (Example 1). Recall that $\beta(k, n)$ denotes the posterior probability that the defendant is guilty when k out of n jurors observe the guilty signal $\mathcal{G}$. It is simple to verify that Assumptions A1 and A2 are satisfied as long as $q_i \in (\beta(1, n), \beta(n-1, n))$ for every juror i. Note that jurors' preferences may differ by quite a margin. For example, for a jury of size $n = 12$, prior $P(I) = P(G) = \frac{1}{2}$, and signal accuracy of $p = \frac{2}{3}$, we have $\beta(1) = 0.000976$ and $\beta(n-1) = 0.999024$.

We are now ready to characterize the sets $\hat{\Gamma}_2, \ldots, \hat{\Gamma}_{n-1}$ and show that our equivalence result holds even with ex-ante weakly undominated strategies.

Proposition 3 *Suppose that Assumptions A1 and A2 hold. Then for every $r = 2, \ldots, n-1$,*
$$\hat{\Gamma}_r = \Gamma_r.$$

Intuitively, assume that Assumptions A1 and A2 hold. Clearly, $\hat{\Gamma}_r \subseteq \Gamma_r$. The proof of Proposition 3 demonstrates the inclusion $\hat{\Gamma}_r \supseteq \Gamma_r$. The proof specifies a set of messages for each juror i of the form $M_i \times T_i$. The sets M_i are determined so that their intersection

contains only one word m^*. Roughly speaking, for every juror i there is a profile of other jurors' strategies that make her pivotal only when their types are either t_{-i}^A or t_{-i}^C (specified in A1). Moreover, for every juror i there is a profile of strategies of her fellow jurors such that i strictly prefers m^* to any other message in M_i. If everyone sends m^*, the mediator transmits a unanimous recommendation to play the action that the equilibrium outcome at hand associates with the vector of types. This specification assures that sending m^* and a truthful type report and then obeying the mediator's recommendation is indeed an undominated strategy, which yields the desired outcome.[17]

Taken together, Propositions 1 and 3 imply that when Assumptions A1 and A2 are satisfied and communication is allowed, all intermediate voting rules are equivalent even if the jurors can only use weakly undominated strategies. Furthermore, it is possible to implement more outcomes under non unanimous voting rules than under unanimous ones.

In this section, we have assumed that the jurors can communicate with an impartial mediator. This assumption was made only to simplify the exposition of the proof of Proposition 3. However, the assumption is not crucial and our equivalence result extends to the case in which an impartial mediator is not available. In fact, by using a communication protocol similar to the one introduced in Gerardi [2004], it is possible to show that if there are at least five jurors and Assumptions A1 and A2 hold, then any outcome in Γ_r, $r = 2, \ldots, n-1$, can be implemented with a sequential equilibrium in undominated strategies of a cheap talk extension with direct communication (for the sake of brevity, we omit the details).

In order to test the robustness of the equivalence result, we have considered sequential equilibria in weakly undominated strategies. However, another route would be to adopt a stronger solution concept. Austen-Smith and Feddersen [2002a, b] use a concept which has the flavor of perfect equilibrium. Specifically, Austen-Smith and Feddersen require that the equilibrium voting behavior of each participant remains optimal even when there is a small probability that the other jurors cast the wrong ballots. One difficulty associated with adopt-

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[17]In Proposition 3 we focus on ex-ante weakly undominated strategies. Clearly, with interim weakly undominated strategies it is easier to implement every outcome in Γ_r, $r = 2, \ldots, n-1$. In particular, Assumption A1 is a sufficient condition.

ing a similar solution concept in our setup comes from the fact that we consider arbitrary forms of communication. Unfortunately, the literature on games with communication has not yet developed enough of a technical apparatus to deal with perfect equilibria. In fact, the characterization of the outcomes induced by perfect equilibria of arbitrary cheap talk extensions is still an open question (the research frontier is probably Dhillon and Mertens [1996], who provide an answer only for two-person games with complete information). We are thus less optimistic about finding general results when concentrating on this particular equilibrium notion at this point in time.

4. DELIBERATIVE VOTING WITH MORE THAN TWO ALTERNATIVES

This section replicates the construction introduced in Section 2 for a general set of alternatives and a general set of allowable actions for each participant.

Consider a group of $n \geqslant 3$ individuals, whom we will refer to as voters, that has to select one of $K \geqslant 2$ alternatives from $\mathcal{A} \equiv \{A_1, A_2, \ldots, A_K\}$. As before, each voter i has a type t_i which is private information. We denote by T_i the set of voter i's types, and assume that T_i is finite. We let $T = \prod_{i=1}^{n} T_i$ denote the set of type profiles. The prior probability distribution over T is denoted by p. As before, we assume that p has full support. A voter's utility depends on the profile of types and the chosen alternative. Formally, for each voter i there exists a function $u_i : \mathcal{A} \times T \to \mathbb{R}$.

A collective choice structure on $\{n, T, \mathcal{A}, \{u_i\}_{i=1}^{n}\}$ constitutes of two elements:

1. The set of available actions. We denote by V_i the actions available to voter i and by $V \equiv V_1 \times V_2 \times \ldots \times V_n$ the set of all possible action profiles.

2. A voting rule, which is a mapping $\psi : V \to \Delta(\mathcal{A})$. Without loss of generality, we assume that any alternative can be selected, that is, $\bigcup_{v \in V} \operatorname{supp} \psi(v) = \mathcal{A}$.

The collective choice structure (V, ψ) defines an analogous Bayesian game $G_{V,\psi}$ to that defined in Section 2.1. Nature selects a profile of types in T according to the probability distribution p, then voters learn their types, after which they vote simultaneously. If the profiles

of types and actions are t and v, respectively, voter i obtains $\sum_{A_k \in \mathcal{A}} \psi\left(A_k|v\right) u_i\left(A_k, t\right)$.

In order to capture outcomes of the voting procedure with communication, we look at cheap talk extensions of $G_{V,\psi}$. Voters exchange messages after learning their types, but before simultaneously casting their votes. We will present the case in which a reliable mediator is handy.[18]

A strategy profile σ of a cheap talk extension of $G_{V,\psi}$ induces an outcome, a mapping γ_σ from the set of types T into the simplex $\Delta(\mathcal{A})$. The vector $\gamma_\sigma(t)$ denotes the probability distribution over collective outcomes when the profile of types is t (and the voters adopt the strategy profile σ). We let $\Gamma_{V,\psi}$ denote the set of outcomes induced by sequential equilibria of cheap talk extensions of $G_{V,\psi}$. As in Section 2.1, we characterize the set $\Gamma_{V,\psi}$ using the notion of communication equilibria. A mapping μ from T into $\Delta(V)$, the set of probability distributions over V, is a communication equilibrium of $G_{V,\psi}$ if and only if the following inequalities hold:

$$\sum_{t_{-i} \in T_{-i}} p\left(t_{-i}|t_i\right) \sum_{v \in V} \mu\left(v|t\right) \sum_{A_k \in \mathcal{A}} \psi\left(A_k|v\right) u_i\left(A_k, t\right) \geqslant$$

$$\sum_{t_{-i} \in T_{-i}} p\left(t_{-i}|t_i\right) \sum_{v \in V} \mu\left(v|t_{-i}, t_i'\right) \sum_{A_k \in \mathcal{A}} \psi\left(A_k|v_{-i}, \delta_i(v_i)\right) u_i\left(A_k, t\right)$$

$$\forall i = 1, \ldots, n, \quad \forall\left(t_i, t_i'\right) \in T_i^2, \quad \forall \delta_i : V_i \to V_i.$$

The set $\Gamma_{V,\psi}$ coincides with the set of outcomes induced by communication equilibria of $G_{V,\psi}$.

$$\Gamma_{V,\psi} = \{\gamma : T \to \Delta(\mathcal{A}) | \exists \text{ a communication equilibrium } \mu \text{ of } G_{V,\psi}$$

$$\text{such that } \gamma\left(A_k|t\right) = \sum_{v \in V} \mu\left(v|t\right) \psi\left(A_k|v\right) \text{ for every } t \in T, \text{ for every } A_k \in \mathcal{A}\}.$$

In the case of two alternatives, a crucial aspect of the equivalence result was the ability to replicate any equilibrium outcome with strategy profiles that were robust to unilateral deviations (via unanimous profiles). We will therefore replicate the construction illustrating

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[18]As before, this assumption is made solely for the sake of presentation simplicity, and could be discarded without affecting the reported results.

the equivalence result for the class of *veto-free* collective choice structures in which no one agent has the power to overturn a choice for all circumstances. Formally,

Definition 2 (Veto-Free Structures) *The collective choice structure (V, ψ) is veto-free if for every A_k in A, there exists a profile $v \in V$ such that for any $i = 1, \ldots, n$, and any $v_i' \in V_i$, $\psi(A_k|v_{-i}, v_i') = 1$.*

That is, the collective choice structure is veto-free if for every alternative, there is a profile of actions that would yield that alternative with probability one, even if one of the committee members deviates. For example, all of the intermediate threshold voting rules discussed in Section 2 are veto-free. The following examples identify most of the well-known multiple alternative voting rules as part of a veto-free collective choice structure (see, e.g., Cox [1987] and references therein).[19]

Examples (Scoring Rules, Alternative Voting, and Condorcet structures)

1. **Generalized Scoring Rules.** A scoring rule is characterized by a set of scores $\{\omega_k\}_{k=1}^{K} \subset \mathbb{R}$. Without loss of generality, we will suppose that there exists a $k^* > 1$ such that $\omega_1 > \omega_2 > \ldots > \omega_{k^*} \geqslant \omega_{k^*+1} \geqslant \ldots \geqslant \omega_K$. Each voter i's action set can be written as $V_i = \{(\alpha_1, \alpha_2, \ldots, \alpha_K) : (\alpha_1, \alpha_2, \ldots, \alpha_K) \text{ is a permutation of } (\omega_1, \omega_2, \ldots, \omega_K)\}$. So a voter reports an allocation of scores to the entire set of candidates. The candidate is then chosen according to:

$$\psi^{score}(A_l|v) = \begin{cases} \frac{1}{|\arg\max_k \sum_{i=1}^{n} v_i(k)|} & l \in \arg\max_k \sum_{i=1}^{n} v_i(k) \\ 0 & \text{otherwise} \end{cases}.$$

The scoring rule $\{\omega_k\}_{k=1}^{K}$ is veto-free if, for example,

$$n\omega_1 - n \; div \; (K-1) \sum_{k=2}^{K} \omega_k - \sum_{k=2}^{n \; mod \; (K-1)} \omega_k > 2(\omega_1 - \omega_K)$$

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[19]We omit some immediate proofs which appear in Gerardi and Yariv [2003].

where $n \ div \ k$ denotes the integer part of $\frac{n}{k}$ and $n \bmod k \equiv n - k \times (n \ div \ k)$, the remainder of n when divided by k.

Some of the scoring rules that are commonly used are in fact veto-free:

Plurality. When $\omega_1 = 1$ and $\omega_2 = \ldots = \omega_K = 0$, the scoring rule is equivalent to the plurality rule. We will denote the equilibria outcomes of the plurality election with communication by $\Gamma^{Plurality}$.

Borda Rule. The scores $\omega_1 = K - 1, \omega_2 = K - 2, \ldots, \omega_K = 0$, correspond to the Borda method of electing an alternative. These scores satisfy the condition for a veto-free structure as well, as long as $n \geqslant 4$. We will denote the equilibria outcomes of the Borda election with communication by Γ^{Borda}.

2. **Alternative Voting.**[20] In the Alternative Voting collective choice structure, each voter reports a strict rank order of the alternatives, that is

$$V_i = \{ \succ \in \mathcal{A} \times \mathcal{A} : \text{for all } k \neq k', A_k \succ A_{k'} \text{ or } A_{k'} \succ A_k \text{ and } \succ \text{ is transitive}\}.$$

The voting rule $\psi^{AV}(v)$ is defined through a recursive process. Top preference alternatives are tallied. The candidate with lowest count is eliminated and the votes are reconsidered as restricted orderings over the remaining $K - 1$ candidates. The process is repeated until one candidate has received half the votes as the most preferred. At each stage, a tie leads to a uniform randomization between the tied candidates. We will denote the set of equilibria corresponding to Alternative Voting with communication by Γ^{AV}.

3. **Condorcet Winner.** In the Condorcet collective choice structure, each voter reports a

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[20] Alternative Voting, commonly referred to as instant runoff voting, is rarely used in the U.S., but has actually been adopted as means of electing local candidates in San Francisco. In addition, it is used to elect the House of Representatives in Australia.

strict rank order of the alternatives as in Alternative Voting:

$$V_i = \{\succ \in \mathcal{A} \times \mathcal{A} : \text{for all } k \neq k', A_k \succ A_{k'} \text{ or } A_{k'} \succ A_k \text{ and } \succ \text{ is transitive}\}.$$

The voting rule $\psi^{Condorcet}(v)$ is defined as follows. For each pair of candidates, it is resolved how many agents preferred each candidate over the other by counting whether they were higher ranked in the reported preference ordering. If any candidate k is preferred to all other candidates, they are declared the winner and $\psi^{Condorcet}(A_k|v) = 1$. If there is no winner, a *top cycle* is determined. A top cycle is a subset of candidates such that each of the members will beat all candidates outside the top cycle in pairwise competition, but not all of the candidates within the top cycle. There are several ways that the literature considers for choosing one candidate as the winner from the top cycle: by uniform randomization, by Alternative Voting within the top cycle, or by choosing the candidate who, in the pair-wise competition she does worst in, loses by the least amount (and randomize upon a tie). All these specifications yield a veto-free structure. We will consider Alternative Voting to take place whenever a runoff vote is necessary and denote the corresponding set of equilibria, when communication is possible, by $\Gamma^{Condorcert}$.

We are now ready to compare the sets $\Gamma_{V,\psi}$ and $\Gamma_{V',\psi'}$ for two different collective choice structures (V, ψ) and (V', ψ'). Proposition 4 shows that all veto-free collective choice structures are equivalent. If voters can communicate, every outcome that can be implemented with a veto-free structure (V, ψ) can also be implemented with a different veto-free structure (V', ψ'). Furthermore, by adopting a non veto-free structure, we cannot enlarge the set of equilibrium outcomes.

Proposition 4 *For any veto-free structures (V, ψ) and (V', ψ'), $\Gamma_{V,\psi} = \Gamma_{V',\psi'}$. Moreover, if $(\tilde{V}, \tilde{\psi})$ is not veto-free, then $\Gamma_{\tilde{V},\tilde{\psi}} \subseteq \Gamma_{V,\psi}$.*

The formal proof of Proposition 4 follows the lines of that of Proposition 1, and is thereby omitted. Intuitively, assume (V, ψ) and (V', ψ') are collective choice structures and assume

that (V, ψ) is veto-free. Consider an outcome implementable with communication under the collective choice structure (V', ψ'). The revelation principle implies that this equilibrium outcome can be implemented with a communication protocol in which voters truthfully reveal their types to an impartial mediator who disperses recommendations to all voters. Each profile of recommended actions corresponds, through ψ', to one of the possible alternatives. Contemplate a modification of this mapping which prescribes to each profile of private reports a profile of recommendations in V that corresponds, via ψ, to the social alternative that would have resulted in the original protocol corresponding to (V', ψ'). Moreover, since (V, ψ) is veto-free, the profile of recommendations can be assumed to be robust to unilateral deviations. In particular, the modified protocol generates an implementable outcome in $\Gamma_{V,\psi}$ coinciding with the one we started with in $\Gamma_{V',\psi'}$. Hence $\Gamma_{V',\psi'} \subseteq \Gamma_{V,\psi}$. If (V', ψ') is veto-free as well, the reverse inclusion is achieved and $\Gamma_{V,\psi} = \Gamma_{V',\psi'}$. Our generalized equivalence result follows.

Since Plurality, Borda, Alternative Voting, and Condorcet collective choice structures are all veto-free when $n \geqslant 4$, it follows from Proposition 4 that they all yield they same set of equilibrium outcomes once communication is introduced. Formally, if we assume that the committee is comprised of at least four members,

Corollary $\Gamma^{Plurality} = \Gamma^{Borda} = \Gamma^{AV} = \Gamma^{Condorcet}$.

Note that similar analysis to that provided in Section 3.1 would assure the generalized equivalence result to hold with unmediated communication and only one round of public deliberations. Moreover, mild restrictions on voters' preferences, as introduced in Section 3.2, would provide the equivalence result when voters use weakly undominated strategies.

5. CONCLUSIONS

The main insight coming out of our current inquiry is that communication between individuals in collective choice scenarios has a fundamental impact on the resulting equilibrium outcomes. In particular, in a generalized voting setup, deliberations render all veto-free rules equivalent with respect to the sequential equilibrium outcomes they generate. This result is

robust to several restrictions on the communication protocols themselves (e.g., one round, public communication), and, under a couple of mild assumptions, when voters are confined to using weakly undominated strategies.

Our analysis illustrates the importance of modeling communication in collective choice environments. It is important to note that when the format of communication is fixed, predictions across voting institutions may, in fact, differ. Therefore, producing a working model of debates and deliberations may be crucial in selecting institutions in particular environments. We view this as an important avenue for future research.

As a particular example, our results suggest the need for future theoretical work to explain the empirical diversity of voting institutions in the jury arena. Indeed, when there are no restrictions on the way jurors can deliberate, the theory implies that predictions of outcomes across a wide range of institutions should be identical.

From a design point of view, our equivalence observation may prove particularly important. Indeed, the plausibility of communication makes the problem of a social planner designing a jury system (or a principal choosing a committee, for that matter) one of equilibrium selection, rather than pure institutional design via the voting rule itself. The analysis in this paper opens a broad set of questions related to the choice of committees (in terms of their size and preference distribution) as well as the specification of communication protocols.

Regarding equilibrium notions, while we succeeded in identifying conditions for the equivalence result to hold when voters are using weakly undominated strategies, it would be interesting to go even further and investigate how institutional outcomes change when voters make mistakes, and realize their fellow jurors may be doing so as well. One way to start such an endeavor would be to look at a stronger equilibrium concept, such as trembling hand perfect equilibrium. Unfortunately, as of yet, there are scarcely any general results in the literature characterizing trembling hand perfect equilibria in games with communication.

6.   Appendix

Proof of Proposition 2

Let γ be an outcome in Γ_r. Consider the following game. In stage 1, all jurors simultaneously send a public message. The set of messages of juror $i = 1, 2$ is $T_i \times [0, 1]$ (i.e., jurors 1 and 2 announce their types and a number in the unit interval). The set of messages of juror $i = 3, \ldots, n$ is equal to T_i (i.e., jurors $3, \ldots, n$ announce their types). In stage 2 the jurors cast their votes.

Consider the following strategy profile. In stage 1, all jurors reveal their types truthfully. Furthermore, both juror 1 and juror 2 randomly select a number in the unit interval, according to the uniform distribution.

Finally, let us describe how the jurors vote in stage 2. Suppose that the vector of types announced in stage 1 is t. Let z_i, $i = 1, 2$, denote the number announced by juror i. Let $\chi : [0, 1]^2 \rightarrow [0, 1]$ denote the following function of z_1 and z_2 :

$$\chi(z_1, z_2) = \begin{cases} z_1 + z_2 & \text{if } z_1 + z_2 \leqslant 1 \\ z_1 + z_2 - 1 & \text{if } z_1 + z_2 > 1 \end{cases}.$$

If $\chi(z_1, z_2) \leqslant \gamma(t)$ all jurors vote to convict. If $\chi(z_1, z_2) > \gamma(t)$ all jurors vote to acquit.

Of course, this strategy profile induces the outcome γ. It is also easy to check that our strategy profile is a sequential equilibrium (consistent beliefs can be derived from any sequence of completely mixed strategy profiles converging to the equilibrium profile). Clearly, a juror does not have a profitable deviation in stage 2 since her vote does not affect the final outcome. By announcing two numbers in the unit interval jurors 1 and 2 perform a jointly controlled lottery which determines how the jurors will vote. Since z_1 is independent of z_2 and uniformly distributed, $\chi(z_1, z_2)$ is also independent of z_2 and uniformly distributed. Thus juror 2 is indifferent between all numbers in $[0, 1]$ (of course, the same argument can be applied to juror 1). Finally, the vector of types announced by the jurors determines which lottery will be used in the second step of the game. Inequalities in (2) guarantee that each juror has an incentive to be sincere provided that all other jurors behave likewise. ∎

Proof of Proposition 3

Fix $r = 2, \ldots, n-1$. To prove Proposition 3 it is enough to show that $\Gamma_r \subseteq \hat{\Gamma}_r$. Assume that Assumptions A1 and A2 hold and fix an outcome γ in Γ_r. Consider the following cheap talk extension. In stage 1 all jurors report their messages simultaneously to the mediator. Each juror i sends a message that has two components. The first component is her type. The second component is an element of the set M_i defined by:

$$M_i = \{0, 1a, 1b, \ldots, (i-1)\,a, (i-1)\,b, (i+1)\,a, (i+1)\,b, \ldots, na, nb\}.$$

In other words, juror i sends a message from the set $T_i \times M_i$. We let $m_i \in M_i$ denote the second component of the message sent by juror i.

For each vector of reports, the mediator selects an action profile in $V = \{a, c\}^n$ according to some probability distribution (specified below) and informs each juror only of her own action. Finally, jurors vote and an alternative is selected according to the voting rule r.

Consider the following specification of the mediator's choice of an action profile. We distinguish between three cases:

- Suppose that $m_j = ia$, for some $i = 1, \ldots, n$ and for each juror $j \neq i$. Let t_{-i} denote the profile of types reported by the jurors different from i (t_{-i} is an element of T_{-i}). In this case, the mediator randomly selects an alternative, A or C, with equal probabilities. First, suppose that alternative A is selected. If $t_{-i} = t^A_{-i}$, the mediator recommends action a to juror i and to the first $n - r$ jurors different from i (i.e., the mediator recommends a to $n - r + 1$ jurors) and action c to every other juror. If $t_{-i} \neq t^A_{-i}$, the mediator recommends action a to all jurors. Suppose now that alternative C is selected. If $t_{-i} = t^C_{-i}$, the mediator recommends action c to juror i and to the first $r - 1$ jurors different from i and action a to every other juror. If $t_{-i} \neq t^C_{-i}$, the mediator recommends action c to all jurors.

- Suppose that for some $i = 1, \ldots, n$, $m_j = ib$, for all jurors $j \neq i$. Let t_i denote the type reported by juror i and t_{-i} the profile of types reported by the jurors different from i.

If $m_i = 0$, the mediator selects (with probability one) the alternative that is optimal for juror i when the profile of types is (t_i, t_{-i}) (if both alternatives are optimal, the mediator selects A). Let $h(t_i, t_{-i})$ denote this alternative. If $m_i \neq 0$, the mediator selects the alternative different from $h(t_i, t_{-i})$. In any case, the mediator recommends to every juror to vote in favor of the chosen alternative.

- Finally, in all other cases, the mediator selects alternatives A and C with probabilities $1 - \gamma(t)$ and $\gamma(t)$, respectively (where t is the profile of types reported by the jurors). The mediator recommends to every juror to vote in favor of the chosen alternative.

Consider now the following strategy for jurors $i = 1, \ldots, n$. Every type t_i reports the message $(t_i, 0)$ and always obeys the mediator's recommendation (even when she reports a message different from $(t_i, 0)$). Let σ_i^* denote this strategy. Of course, the strategy profile $(\sigma_1^*, \ldots, \sigma_n^*)$ induces the outcome γ. It is also easy to show that we can find a system of beliefs $(\beta_1^*, \ldots, \beta_n^*)$ such that the assessment $((\sigma_1^*, \ldots, \sigma_n^*), (\beta_1^*, \ldots, \beta_n^*))$ constitutes a sequential equilibrium of our game. It is enough to take a sequence of completely mixed strategy profiles which converges to $(\sigma_1^*, \ldots, \sigma_n^*)$ and such that for each juror $i = 1, \ldots, n$, deviations to messages with the second component in the set $\{1a, \ldots, (i-1)a, (i+1)a, \ldots, na\}$ are much less likely than other deviations. This implies that in every information set a juror assigns probability zero to the event that her vote is pivotal (we omit the details). Sequential rationality of the assessment $((\sigma_1^*, \ldots, \sigma_n^*), (\beta_1^*, \ldots, \beta_n^*))$ trivially follows.

To complete our proof, we need to show that for each juror $i = 1, \ldots, n$, the strategy σ_i^* is ex-ante weakly undominated. Of course, a strategy for juror i specifies for each type t_i the message that t_i sends and an action for every pair of recommendations and message (even the messages that were not sent). However, for our purposes it is enough to consider the reduced representation and restrict attention to the actions corresponding to the message actually sent.

Let $S(t_i)$ denote the set of pure strategies of type t_i in which t_i does not always obey the mediator's recommendation. Denote by $S'(t_i)$ the set of pure strategies of t_i in which t_i sends

a message different from $(t_i, 0)$ and then obeys the mediator's recommendation. Consider any strategy σ_i of juror i different from σ_i^*. At least one of the following two alternatives is true: (i) there exists a type $\hat{t}_i$ such that $\sigma_i \left(\hat{t}_i \right)$ assigns positive probability to a strategy in the set $S \left(\hat{t}_i \right)$; (ii) there exists a type $\hat{t}_i$ such that $\sigma_i \left(\hat{t}_i \right)$ assigns positive probability to a strategy in the set $S' \left(\hat{t}_i \right)$.

Start with case (i). Consider the strategy profile of jurors different from i in which every type t_j of juror $j \neq i$ sends the message (t_j, ia) and then obeys the mediator's recommendation. It follows from assumption A1 that against this strategy profile, type $\hat{t}_i$ strictly prefers the strategy $\sigma_i^* \left(\hat{t}_i \right)$ to the strategy $\sigma_i \left(\hat{t}_i \right)$. Moreover, assumption A1 also implies that for every other type t_i the strategy $\sigma_i^* \left(t_i \right)$ is weakly better than the strategy $\sigma_i \left(t_i \right)$.

For case (ii), consider the strategy profile in which every type t_j of juror $j \neq i$ sends the message (t_j, ib) and then is obedient. Denote this profile by σ'_{-i} and consider type $\hat{t}_i$. It follows from assumption A2 that against σ'_{-i}, the strategy $\sigma_i^* \left(\hat{t}_i \right)$ is strictly better than any pure strategy in which $\hat{t}_i$ sends a message $(t_i, 0)$, where $t_i \neq \hat{t}_i$. Furthermore, assumption A1 implies that against σ'_{-i}, $\sigma_i^* \left(\hat{t}_i \right)$ is strictly better than any pure strategy in which $\hat{t}_i$ sends a message (t_i, m_i), where $m_i \neq 0$ and $t_i \in T_i$. Of course, against σ'_{-i} disobedience is not beneficial. Thus, $\hat{t}_i$ strictly prefers $\sigma_i^* \left(\hat{t}_i \right)$ to $\sigma_i \left(\hat{t}_i \right)$. Finally, assumptions A1 and A2 also imply that against σ'_{-i} any other type t_i weakly prefers $\sigma_i^* \left(t_i \right)$ to $\sigma_i \left(t_i \right)$. ∎

REFERENCES

[1] Austen-Smith, D. and Banks, J. [1996], "Information Aggregation, Rationality, and The Condorcet Jury Theorem," *American Political Science Review*, Volume 90, pages 34-45.

[2] Austen-Smith, D. and Feddersen, T. [2002a], "Deliberation and Voting Rules," The Center for Mathematical Studies in Economics and Management Science at Northwestern University, Discussion Paper Number 1359.

[3] Austen-Smith, D. and Feddersen, T. [2002b], "The Inferiority of Deliberation Under Unanimity," The Center for Mathematical Studies in Economics and Management Science at Northwestern University, Discussion Paper Number 1360.

[4] Blinder, A. S., Morgan, J. [2005], "Are Two Heads Better than One?: An Experimental Analysis of Group versus Individual Decision-making," *Journal of Money, Credit and Banking*, Volume 37, pages 789-811.

[5] Condorcet, Marquis de. [1785], *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la probabilité des voix*. Paris: De l'imprimerie royale, translated in 1976 to "Essay on the Application of Mathematics to the Theory of Decision-Making." in *Condorcet: Selected Writings*, ed. Keith M. Baker. Indianapolis, IN: Bobbs-Merrill.

[6] Coughlan, P. [2000], "In Defense of Unanimous Jury Verdicts: Mistrials, Communication, and Strategic Voting," *American Political Science Review*, Volume 94, pages 375-393.

[7] Cox, G. W. [1987], "Electoral Equilibrium under Alternative Voting Institutions," *American Journal of Political Science*, Volume 31, pages 82-108.

[8] Dhillon, A. and Mertens, J. F. [1996], "Perfect Correlated Equilibria," *Journal of Economic Theory*, Volume 68, pages 279-302.

[9] Doraszelski, U., Gerardi, D., and Squintani, F. [2003], "Communication and Voting with Double-Sided Information," *Contributions to Theoretical Economics*, Volume 3, Article 6.

[10] Feddersen, T. J. and Pesendorfer, W. [1996], "The Swing Voter's Curse," *American Economic Review*, Volume 86, pages 408-424.

[11] Feddersen, T. J. and Pesendorfer, W. [1997], "Voting Behavior and Information Aggregation in Elections with Private Information," *Econometrica*, Volume 65, pages 1029-58.

[12] Feddersen, T. J. and Pesendorfer, W. [1998], "Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting," *The American Political Science Review*, Volume 92, pages 23-35.

[13] Forges, F. [1986], "An Approach to Communication Equilibria," *Econometrica*, Volume 54, pages 1375-85.

[14] Forges, F. [1990], "Universal Mechanisms," *Econometrica*, Volume 58, pages 1341-1364.

[15] Fudenberg, D. and J. Tirole [1991], *Game Theory*. Cambridge, MA: The MIT Press.

[16] Gerardi, D. [2004], "Unmediated Communication in Games with Complete and Incomplete Information," *Journal of Economic Theory*, Volume 114, pages 104-131.

[17] Gerardi, D. and Yariv, L. [2005], "Information Acquisition in Committees," mimeo.

[18] Gerardi, D. and Yariv, L. [2003], "Putting Your Ballot Where Your Mouth Is: An Analysis of Collective Choice with Communication," UCLA Working Papers, Discussion Paper Number 827.

[19] Goeree, J. K. and Yariv, L. [2006], "An Experimental Study of Jury Deliberation," mimeo.

[20] Guarnaschelli, S., McKelvey, R. C., and Palfrey, T. R. [2000], "An Experimental Study of Jury Decision Rules," *American Political Science Review*, Volume 94, pages 407-423.

[21] Hastie, R., Penrod, S. D., and Pennington, N. [1983], *Inside the Jury*, Harvard University Press.

[22] Kalven, H. and Zeisel, H. [1966], *The American Jury*, Little Brown & Company.

[23] Matthews, S. and Postlewaite, A. [1989], "Pre-play Communication in Two-Person Sealed-Bid Double Auctions," *Journal of Economic Theory*, Volume 48, pages 238-263.

[24] McLean, R. and Postlewaite, A. [2002], "Information Size and Incentive Compatibility," *Econometrica*, Volume 70, pages 2421-2453.

[25] Myerson, R. B. [1982], "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," *Journal of Mathematical Economics*, Volume 10, pages 67-81.

[26] Myerson, R. B. [1991], *Game Theory: Analysis of Conflict.* Cambridge, MA: Harvard University Press.

[27] Myerson, R. [1998], "Extended Poisson Games and the Condorcet Jury Theorem," *Games and Economic Behavior*, Volume 25, pages 111-131.

[28] Persico, N. [2004], "Committee Design with Endogenous Information," *The Review of Economic Studies*, Volume 71, pages 165-94.